Centerpoints: Link between Convex Geometry and Optimization

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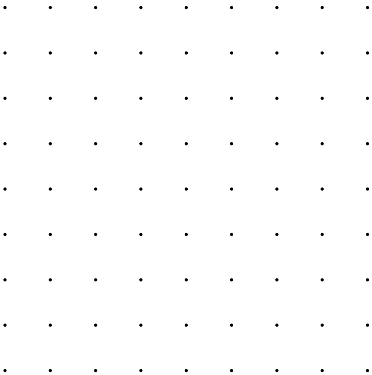

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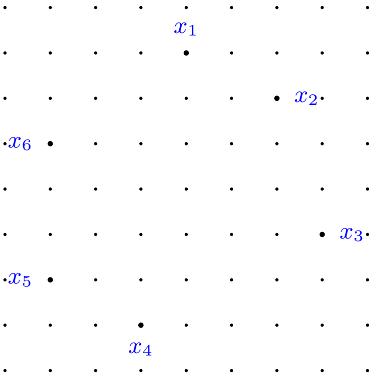

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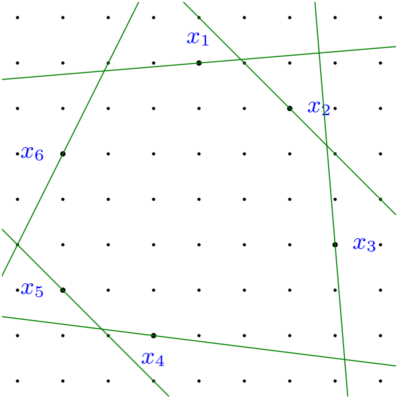

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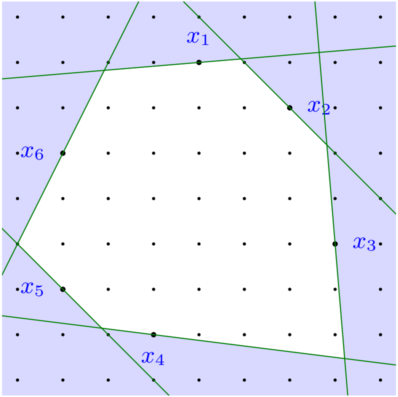

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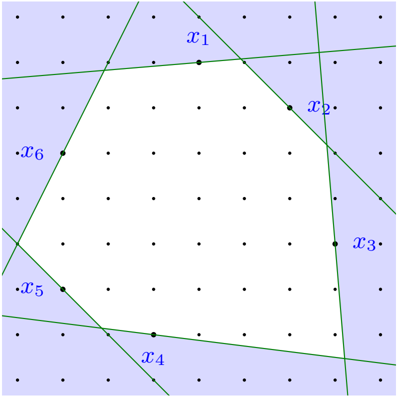

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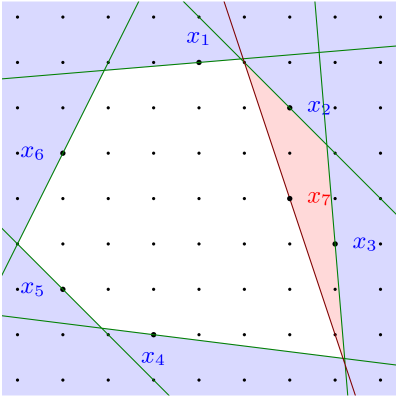

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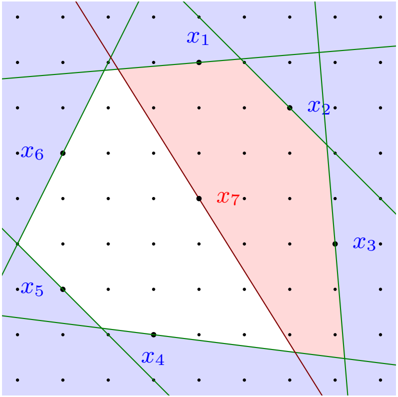

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Measure of Progress

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Set over which we optimize:
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For $x \in \mathbb{R}^n$ and $u \in \mathcal{S}^{n-1}$ we denote

$$H^{\geq}(u,x) := \left\{ y \in \mathbb{R}^n \mid u^T(y-x) \geq 0 \right\}.$$

Definition

A *centerpoint* w.r.t. S and μ is defined as an optimal solution $x^\star$ of

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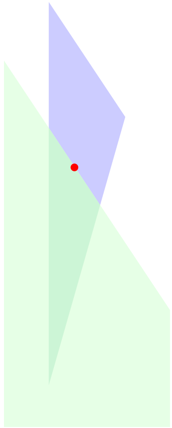

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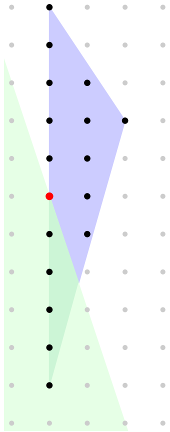

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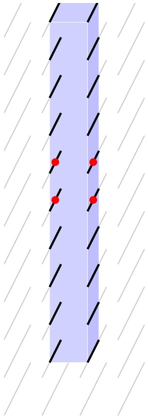

μ mixed-integer measure;
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Median

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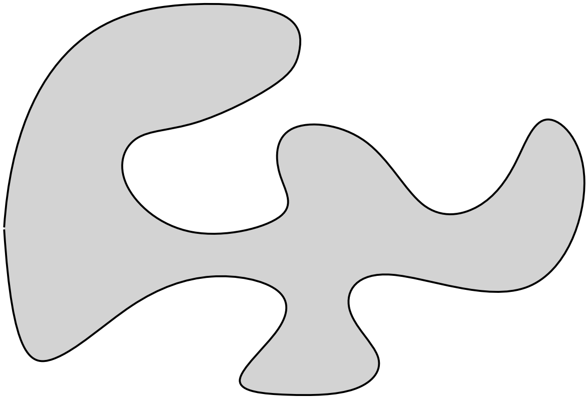
Winternitz' measure of symmetry

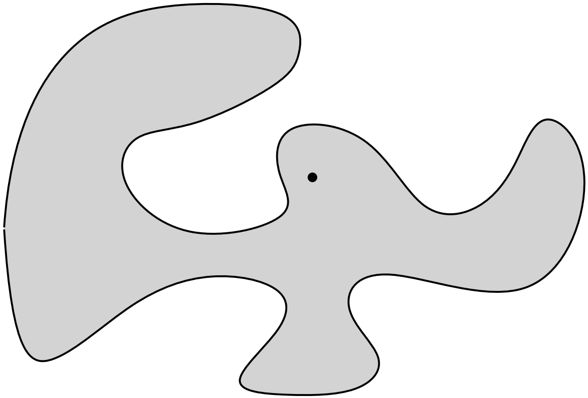
$S = \mathbb{R}^n$ and μ be the uniform measure on a convex set K: $\mu(C) = \mathrm{vol}(C \cap K)\,\mathrm{vol}(K)^{-1}$. Then

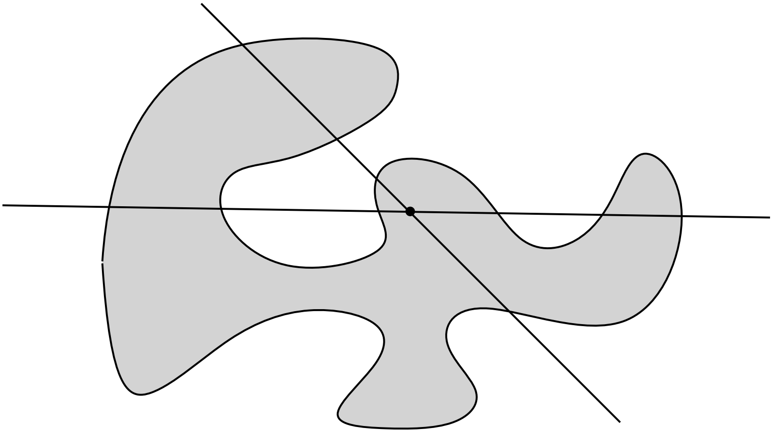
$$\min_{u \in \mathcal{S}^{n-1}} \mu(H^{\geq}(u, x^\star)) = \frac{1}{2}.$$

if and only if

$$K - x^\star = x^\star - K \qquad \text{(Funk 1915)}$$







Centerpoints and the Helly-Number

Let $S \subset \mathbb{R}^n$ and $\mathcal{K} := \{S \cap K \mid K \subset \mathbb{R}^n \text{ convex } \}$. The *Helly-Number* $h(S) \in \mathbb{Z}_+$ is defined as the minimal number such that: For any $\{K_1, \ldots, K_m\} \subset \mathcal{K}$, if

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then there exists $\{i_1, \ldots, i_h\} \subset \{1, \ldots, m\}$ such that

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Theorem [Helly 1913, Doignon 1973, Hoffman 1979]

$$h(\mathbb{Z}^n \times \mathbb{R}^d) = 2^n(d+1)$$

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Theorem [Basu–Oertel 2015]

Let $S \subseteq \mathbb{R}^n$ be a closed subset and let μ be such that $\mu(\mathbb{R}^n \setminus S) = 0$. If $h(S) < \infty$, then

$$\max_{x \in S} \inf_{u \in \mathcal{S}^{n-1}} \mu(S \cap H^{\geq}(u, x)) \geq h(S)^{-1}.$$

Centerpoints in $\mathbb{R}^d$

Let $S = \mathbb{R}^d$ and let μ be a uniform measure w.r.t. a closed convex set $K \subset \mathbb{R}^d$. Let $x^\star$ denote its corresponding centerpoint.

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Theorem [Grünbaum 1960]

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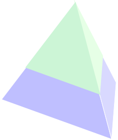

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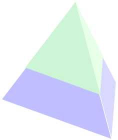

Thus, $\mu(H^{\geq}(u, x^\star)) \geq e^{-1}$.

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Theorem [Basu–Oertel 2015]

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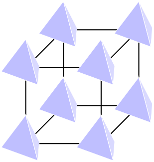

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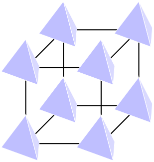

Conjecture [Basu–Oertel 2015]

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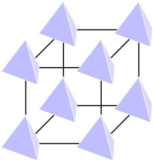

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Theorem [Basu–Oertel 2015]

Let ω be the lattice width of K. If $\omega \geq c\, 2^{O(n)}$ such that $e^{-\frac{1}{c}-1} + e^{-\frac{2}{c}} - 1 \geq 2^{-n-1}$, then

$$\mu_K(H^{\geq}(u, x^\star)) \geq \frac{1}{2^n} \left(\frac{d}{d+1}\right)^d.$$

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g(x) \leq 0, \\ & x \in S. \end{aligned}$$

Let $S \subset \mathbb{R}^n$ be closed.

$f : \mathbb{R}^n \mapsto \mathbb{R}$,

$g : \mathbb{R}^n \mapsto \mathbb{R}^m$: convex, given by a first-order function oracles, queried on a point $x \in S$ the oracle returns $f(x)$ and $h \in \partial f(x)$ or g respectively.

We assume $\exists\, B \in \mathbb{N}$ such that $\{x \in \mathbb{R}^n \mid g(x) \leq 0\} \subset [-B, B]^n$.

f is Lipschitz continuous, with Lipschitz constant L.

Let K be a convex, compact body and

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General cutting plane scheme

- Let $P_0 := [0, B)^{n+d} \cap (\mathbb{Z}^n \times \mathbb{R}^d)$.
- For $i \leq N$
  - Compute centerpoint x_i w.r.t. μ_{P_i} and S.
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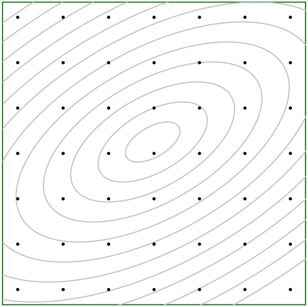

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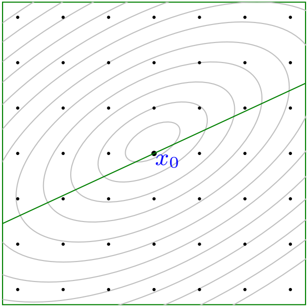

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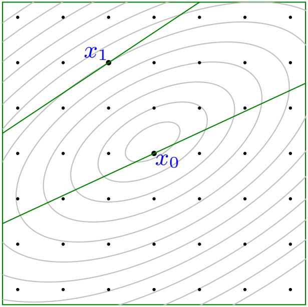

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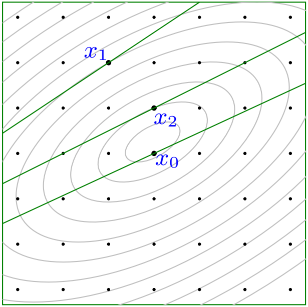

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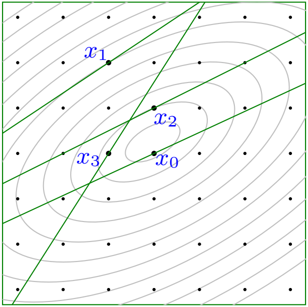

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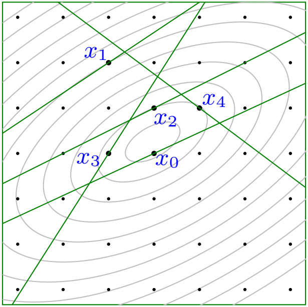
- For $i \leq N$
  - Compute centerpoint x_i w.r.t. μ_{P_i} and S.
  - Let $h \in \partial f(x_i)$.
  - Define $P_{i+1} = P_i \cap H^{>}(h, x_i)$.
- Return $\bar{x} := \text{argmin}\, f(x_i)$.



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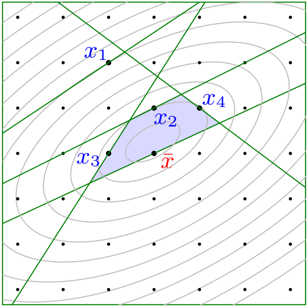

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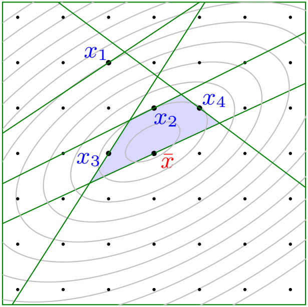

Upper bound

Let $\delta > 0$, and $k^\star \leq \min_{u \in \mathcal{S}^{n+d-1}} \mu_K(H^{\geq}(u, x_S^\star))$ for all compact convex sets K and corresponding centerpoints $x_S^\star$.

After N iterations $\mu_{P_0}(P_N) \leq (1 - k^\star)^N$. Thus, for $N \geq \left\lceil \log_{\frac{1}{1-k^\star}} \left(\frac{B^{n+d}}{\delta} \right) \right\rceil$

$$\mu_{P_0}(P_N) \leq \delta.$$

If $\delta \leq \epsilon / L$, then $f(\bar{x}) - \min_{x \in \mathbb{Z}^n \times \mathbb{R}^d} f(x) \leq \epsilon$.

Number of Function Oracle Calls

S	**Upper Bound**	**Lower Bound**
$\mathbb{R}^d$	$\left\lceil \log_{\frac{e}{e-1}} \left(\frac{B^d}{\delta}\right) \right\rceil$	$\left\lceil \log_2 \left(\frac{B^d}{\delta}\right) \right\rceil - 1$ (Yudin & Nemirovsky)
$\mathbb{Z}^n$		
$\mathbb{Z}^n \times \mathbb{R}^d$		

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$\mathbb{Z}^n \times \mathbb{R}^d$	$O\left(2^n(d+1)\log_2\left(\frac{B^{n+d}}{\delta}\right)\right)$	$\Omega\left(2^n\left(\log_2\frac{B^d}{\delta}\right)\right)$	(Basu & Oertel)

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Let $S = \mathbb{Z}^n \times \mathbb{R}^d$ and let μ be the uniform measure with respect to $K \cap S$, where K is a polytope.

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- For fixed n and d:
  By random sampling, we can approximate centerpoints with a high probability. (using Vapnik-Chervonenkis theory)

Question

Let $S = \mathbb{Z} \times \mathbb{R}^d$, $K \in [-1, 1] \times \mathbb{R}^d$ be a convex body and let μ denote the uniform measure on $K \cap S$.

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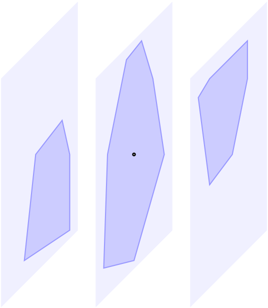

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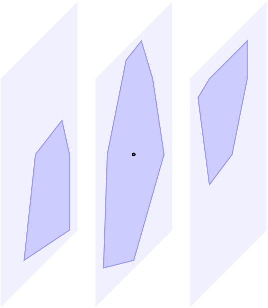

THANKS