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Abstract

Theoretical studies usually assume that all agents are self-interested. On the other hand, we consider that there are some partially honest agents in the sense of Dutta and Sen[8]. If all agents are self-interested, Maskin monotonicity and no veto power are together sufficient for double implementation in Nash equilibria and undominated Nash equilibria (Jackson et al.[16], Tatamitani[39], and Yamato [42][43]). We show that if at least one agent is partially honest, no veto power is sufficient for double implementation with partially honest agents. Therefore, we no longer need Maskin monotonicity as a necessary condition of double implementability. Moreover, we show that if at least two agents are partially honest, unanimity is sufficient for double implementation with partially honest agents.

JEL Classification: C72, D71, D78

Key words: Partial honesty, double implementation, Unanimity, No veto power, Social choice correspondence

1 Introduction

The theory of mechanism design aims to identify a mechanism achieving a socially goal across a domain of agents' preferences. Theoretical studies usually assume that all agents are self-interested: an agent is self-interested if she only cares about the outcome(s) obtained from the mechanism.

On the other hand, experimental studies observe that some agents have intrinsic preferences for honesty. A bunch of studies discuss the issue of implementation when agents have intrinsic preferences for honesty.[1] Dutta and Sen [8] construct a mechanism in which each agent reports a preference profile and an outcome. Under their mechanism, Dutta

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[1]See for example, Diss et al. [3], Doghmi [4], Doghmi and Ziad [5] [6], Dutta and Sen [8], Hagiwara et al.[11], Kartik et al. [19], Kimya [21], Korpela [22], Lombardi and Yoshihara [25] [26] [27] [28], Matsushima [29] [30], Mukherjee and Muto [32], Núñes and Laslier[33], Ortner [34], and Saporiti [37].

and Sen [8] assume that at least one agent is partially honest: an agent is partially honest whenever, if the outcomes obtained from the mechanism are indifferent, she prefers reporting the true preference profile; otherwise, she prefers reporting a message inducing more preferred outcome. They prove that if there are at least three agents and at least one agent is partially honest, then every social choice correspondence (SCC) satisfying no veto power can be implemented in Nash equilibria with partially honest agents by their mechanism.[2] Also, Kimya [21] establishes that if there are at least three agents and all agents are partially honest, then every SCC satisfying unanimity can be implemented in Nash equilibria with partially honest agents by Dutta and Sen's mechanism. He mentions that his result is still valid if at least two agents are partially honest.

However, for Dutta and Sen's mechanism, the set of undominated Nash equilibrium outcomes may be strictly smaller than the set of Nash equilibrium outcomes.[3] Thus, Dutta and Sen's mechanism may not implement an SCC with partially honest agents if agents do not use weakly dominated strategies. Then, is it sufficient to design a mechanism that implements an SCC in undominated Nash equilibria with partially honest agents? Our answer is negative because laboratory evidence casts doubt on the assumption that agents adopt dominant strategies. In pivotal mechanism experiments in which for each agent, telling her true value is a dominant strategy, Attiyeh et al. [1] and Kawagoe and Mori [20] observed that less than half of subjects adopted dominant strategies. Moreover, in second price auction experiments in which for each agent, bidding her true value is a dominant strategy, Kagel et al. [18], Kagel and Levin [17], and Harstad [12] observed that most bids did not reveal true values. It was not obvious whether or not each agent adopted dominant strategies. Thus, it is desirable to construct mechanisms that are applicable not only when agents use dominant strategies but also when they do not. As Cason et al. [2] point out, on the other hand, the high rate of the observed non-dominant strategy outcomes were Nash equilibria in their experiments. Therefore, although subjects frequently played Nash equilibria, there was no guarantee that they did not use weakly dominated strategies. Then, we are concerned with the design of a mechanism that doubly implements an SCC in Nash equilibria and undominated Nash equilibria with partially honest agents.

Previous studies show that if there are at least three agents and all agents are self-interested, Maskin monotonicity and no veto power are together sufficient for double implementation (Jackson et al.[16], Tatamitani[39], and Yamato [42][43]). On the other hand, we consider that there are some partially honest agents in the sense of Dutta and Sen [8]. We show that if there are at least three agents and at least one agent is partially honest in the sense of Dutta and Sen [8], no veto power is still sufficient for double implementation with partially honest agents (Theorem 2). Therefore, we no longer need Maskin monotonicity as a necessary condition of double implementability. Moreover, we show that if there are at least three agents and at least two agents are partially honest in the sense of Dutta and Sen [8], unanimity is still sufficient for double implementation

[2] Lombardi and Yoshihara [26] provide a characterization of implementation in Nash equilibria with partially honest agents if there are at least three agents and at least one agent is partially honest.

[3] Yamato[43] provides an example that in a mechanism used by Maskin[31], the set of undominated Nash equilibrium outcomes may be strictly smaller than the set of Nash equilibrium outcomes. Also, we can easily show that, in Dutta and Sen's mechanism, the set of undominated Nash equilibrium outcomes with partially honest agents may be strictly smaller than the set of Nash equilibrium outcomes with partially honest agents even if two agents are partially honest. See Yamato[43] and Example 4.

with partially honest agents (Theorem 3). Hence, more social choice correspondences can be doubly implemented with partially honest agents if at least two agents are partially honest since unanimity is weaker than no veto power. As described above, we show that the results of Dutta and Sen [8] and Kimya [21] are still valid for double implementation with partially honest agents.

To show the practical value of our main results, we examine the implementability in problems of allocating an infinitely divisible resource, coalitional games, general problems of one-to-one matching, and voting games. Since all SCCs considered here violate Maskin monotonicity, the SCCs cannot be doubly implemented in the standard setting. But the SCCs can be doubly implemented with partially honest agents.

This paper is organized as follows. Section 2 presents the theoretical framework and outlines the basic model. Section 3 includes assumptions on partially honest agents. Section 4 reports our main results about double implementation with partially honest agents. Section 5 discusses implications. Section 6 provides concluding remarks. Appendix includes the proof of Theorem 2 and Theorem 3.

2 Notation

Let A be the arbitrary set of outcomes, and $N = \{1, ..., n\}$ be the set of agents, with generic element i. Let R_i be a preference ordering for agent $i \in N$ over A and $\Re_i$ be the set of all preference orderings for agent $i \in N$. Let P_i and I_i be the asymmetric and symmetric components of $R_i \in \Re_i$, respectively. Let $R = (R_1, ..., R_n)$ be a preference profile and $\Re = \times_{i \in N} \Re_i$ be the set of all preference profiles. Let $\mathcal{D} = \times_{i \in N} \mathcal{D}_i \subseteq \Re$ where $\mathcal{D}_i \subseteq \Re_i$ for each $i \in N$ be a *domain.*

A *social choice correspondence (SCC)* is a mapping $F : \mathcal{D} \twoheadrightarrow A$ that specifies a non-empty subset $F(R) \subseteq A$ for each $R \in \mathcal{D}$. Given an SCC F, an outcome $a \in A$ is *F-optimal at $R \in \mathcal{D}$* if $a \in F(R)$.

A *mechanism* Γ consists of a pair (M, g) where $M = \times_{i \in N} M_i$, M_i is the *message* (or *strategy*) *space of agent* $i \in N$, and $g : M \longrightarrow A$ is the *outcome function* mapping each message profile $m \in M$ into an outcome $g(m) \in A$. For each $i \in N$ and each $m \in M$, let $m_{-i} \in M_{-i} = \times_{j \neq i} M_j$ be the message profile except agent $i \in N$. That is, $m_{-i} = (m_1, ..., m_{i-1}, m_{i+1}, ..., m_n)$. The message profile $m \in M$ is also written as (m_i, m_{-i}). Given $m \in M$ and $m'_i \in M_i$, (m'_i, m_{-i}) is the message profile $(m_1, ..., m_{i-1}, m'_i, m_{i+1}, ..., m_n)$ obtained after the replacement of $m_i \in M_i$ by $m'_i \in M_i$.

3 Assumptions on Partially Honest Agents

The literature on mechanism design usually assumes that each agent only cares about the outcome(s) obtained from the mechanism. However, some recent studies assume that at least some agents may have *intrinsic preferences for honesty.* Dutta and Sen [8] construct a mechanism in which each agent reports a preference profile and an outcome. Under their mechanism, Dutta and Sen [8] assume that at least one agent is partially honest: an agent is partially honest whenever, if the outcomes obtained from the mechanism are indifferent, she prefers reporting the true preference profile; otherwise, she prefers reporting a message inducing more preferred outcome. Lombardi and Yoshihara [26] extend Dutta and Sen's notion of partially honesty by introducing a truth-telling correspondence for any mechanism. We follow Lombardi and Yoshihara [26].

Let an SCC F be given. For each $i \in N$ and each mechanism Γ, a *truth-telling correspondence* T_i^Γ is a mapping $T_i^\Gamma : \mathcal{D} \twoheadrightarrow M_i$ that specifies a non-empty set of truth-telling messages $T_i^\Gamma(R) \subseteq M_i$ for each $R \in \mathcal{D}$. Given a mechanism Γ, a truth-telling correspondence T_i^Γ, and $R \in \mathcal{D}$, we say that agent $i \in N$ *behaves truthfully* at $m \in M$ if and only if $m_i \in T_i^\Gamma(R)$.

We focus on following mechanisms and truth-telling correspondences.

Example 1. In this paper, we focus on mechanisms in which each agent reports a preference profile and a supplemental message. For each $i \in N$, the message space of agent $i \in N$ consists of $M_i = \mathcal{D} \times S_i$, where S_i denotes the set of supplemental messages.

For each $i \in N$, $m_i = (R^i, s^i)$ is a truth-telling message if and only if $R^i = R$. Then, a truth-telling correspondence is defined by $T_i^\Gamma(R) = \{R\} \times S_i$ for each $i \in N$ and each $R \in \mathcal{D}$.

For each $i \in N$, each $R \in \mathcal{D}$, each mechanism Γ, and each truth-telling correspondence T_i^Γ, agent i's preference ordering $\succsim_i^R$ over M at $R \in \mathcal{D}$ is defined below.[4]

Definition 1. An agent $i \in N$ is *partially honest* if for each $R \in \mathcal{D}$ and each $(m_i, m_{-i}), (m_i', m_{-i}) \in M$, the following properties hold:

(1) If $m_i \in T_i^\Gamma(R)$ and $m_i' \notin T_i^\Gamma(R)$ and if $g(m_i, m_{-i}) R_i g(m_i', m_{-i})$, then $(m_i, m_{-i}) \succ_i^R (m_i', m_{-i})$.

(2) In all other cases, $(m_i, m_{-i}) \succsim_i^R (m_i', m_{-i})$ if and only if $g(m_i, m_{-i}) R_i g(m_i', m_{-i})$.

The first part of the definition captures the agent's limited preference for honesty - she strictly prefers (m_i, m_{-i}) to (m_i', m_{-i}) when she reports truthfully in (m_i, m_{-i}) but not in (m_i', m_{-i}) if $g(m_i, m_{-i})$ is at least as good as $g(m_i', m_{-i})$.[5]

Definition 2. An agent $i \in N$ is *self-interested* if for each $R \in \mathcal{D}$ and each $(m_i, m_{-i}), (m_i', m_{-i}) \in M$,

$$(m_i, m_{-i}) \succsim_i^R (m_i', m_{-i}) \text{ if and only if } g(m_i, m_{-i}) R_i g(m_i', m_{-i}).$$

Since self-interested agents care only about the outcomes obtained from the mechanism, their preference orderings over M are straightforward to define.

The traditional literature on mechanism design usually makes the following assumption:

[4] Let $\succ_i^R$ and $\sim_i^R$ be the asymmetric and symmetric components of $\succsim_i^R$, respectively.

[5] There are other definitions of preferences for honesty. For instance, Matsushima [30] investigates a mechanism in which each agent reports an outcome many times. Under his mechanism, Matsushima [30] assumes that at least one agent is white lie averse: agent is white lie averse whenever, as long as her lie does not influence the outcome choice and the monetary transfer to her, she likes telling a socially desirable outcome as many times as possible. Moreover, Mukherjee and Muto [32] design a mechanism in which each agent reports her own preference ordering many times. Under their mechanism, Mukherjee and Muto [32] assume that all agents are partially honest: an agent is partially honest whenever, if the outcomes obtained from the mechanism are indifferent, she prefers reporting her own true preference ordering as many times as possible.

Assumption 0. There is no partially honest agent in N. That is, all agents are self-interested.

In contract to the traditional literature, Dutta and Sen [8] and Lombardi and Yoshihara [23] [26] study the following assumption:

Assumption 1. There exists *at least one* partially honest agent in N. The mechanism designer knows that there exists at least one partially honest agent in N, though she does not know their identities or their exact number.

Moreover, Korpela [22] and Saporiti [37] introduce the following assumption:

Assumption *n*. There are n partially honest agents in N. That is, all agents are partially honest.

Hagiwara et al.[11] consider the following assumption:

Assumption 2. There are *at least two* partially honest agents in N. The mechanism designer knows that there are at least two partially honest agents in N, though she does not know their identities or their exact number.

Clearly, Assumption 2 is stronger than Assumption 1, but significantly weaker than Assumption n.

We introduce our formal definitions of double implementation with partially honest agents under Assumption $k \in \{1, 2, n\}$. For each $k \in \{1, 2, n\}$, let $\mathcal{H}^k = \{S \subseteq N \mid |S| \geq k\}$. For each $R \in \mathcal{D}$ and each $H \in \mathcal{H}^k$, let $\succsim^{R,H} = (\succsim_1^{R,H}, ..., \succsim_n^{R,H})$ be the preference profile over M such that for each $i \in H$, $\succsim_i^{R,H}$ is defined by Definition 1 and for each $i \in N \backslash H$, $\succsim_i^{R,H}$ is defined by Definition 2.

Let $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ be a *game with partially honest agents* induced by a mechanism Γ, a truth-telling correspondence T_i^Γ for each $i \in N$, and a preference profile $\succsim^{R,H}$. A message profile $m \in M$ is a *Nash equilibrium with partially honest agents in* $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ if for each $i \in N$ and each $m_i' \in M_i$, $(m_i, m_{-i}) \succsim_i^{R,H} (m_i', m_{-i})$. The set of Nash equilibria with partially honest agents in $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ is denoted by $NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$. Also, the set of Nash equilibrium outcomes with partially honest agents in $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ is denoted by $NE_A(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H}) = \{a \in A | \exists m \in NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ with $g(m) = a\}$.

A message $m_i \in M_i$ is *weakly dominated by* $\tilde{m}_i \in M_i$ *at* $\succsim_i^{R,H}$ if $(\tilde{m}_i, m_{-i}) \succsim_i^{R,H} (m_i, m_{-i})$ for each $m_{-i} \in M_{-i}$ and $(\tilde{m}_i, m_{-i}) \succ_i^{R,H} (m_i, m_{-i})$ for some $m_{-i} \in M_{-i}$. A message $m_i \in M_i$ is *undominated at* $\succsim_i^{R,H}$ if it is not weakly dominated by any message in M_i *at* $\succsim_i^{R,H}$. A message profile $m \in M$ is an *undominated Nash equilibrium with partially honest agents in* $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ if for each $i \in N$, $m_i \in M_i$ is undominated at $\succsim_i^{R,H}$ and $m \in M$ is a Nash equilibrium with partially honest agents in $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$. The set of undominated Nash equilibria with partially honest agents in $(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$ is denoted by $UNE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$. Note that $UNE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H}) \subseteq NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$. Also, the set of undominated Nash equilibrium outcomes with partially honest agents in

$(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H})$ is denoted by $UNE_A(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H}) = \{a \in A | \exists m \in UNE(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H}$ $)$ with $g(m) = a\}$.

Under Assumption 1 or Assumption 2, the mechanism designer knows that there are partially honest agents in N but does not know who these agents are. Hence, the mechanism designer needs to cover all feasible cases of partially honest agents to her knowledge. To enable the mechanism designer to implement an SCC with partially honest agents, we amend the standard definition of implementation as follows:

Definition 3. Under Assumption $k \in \{1, 2, n\}$, a mechanism Γ *partially honest doubly implements an SCC F in Nash equilibria and undominated Nash equilibria* if for each $R \in \mathcal{D}$ and each $H \in \mathcal{H}^k$, $F(R) = NE_A(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H}) = UNE_A(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H})$.

4 Main Results

First, we review a previous result on double implementation under Assumption 0. Maskin [31] introduces the following properties of SCC's.

For each $i \in N$, each $R_i \in \mathcal{D}_i$, and each $a \in A$, let $L(R_i, a) = \{b \in A \mid aR_ib\}$ be the *lower contour set of $a \in A$ for $i \in N$ at $R_i \in \mathcal{D}_i$.*

An SCC F satisfies *Maskin monotonicity* if for each R, $R' \in \mathcal{D}$ and each $a \in F(R)$, if for each $i \in N$, $L(R_i, a) \subseteq L(R'_i, a)$, then $a \in F(R')$. Maskin monotonicity requires that if an outcome $a \in A$ is F-optimal at some preference profile and the profile is then altered so that, in each agent's ordering, the outcome a does not fall below any outcome that was not below before, then the outcome a remains F-optimal at the new profile.

Definition 4. An SCC F satisfies *no veto power* if for each $i \in N$, each $R \in \mathcal{D}$, and each $a \in A$ if for each $j \neq i$, $L(R_j, a) = A$, then $a \in F(R)$.

No veto power says that if an outcome $a \in A$ is at the top of $(n-1)$ agents' preference orderings, then the last agent cannot prevent the outcome a from being F-optimal at the preference profile.

Previous studies show that Maskin monotonicity and no veto power are together sufficient for double implementation under Assumption 0.

Theorem 1. *(Jackson et al.[16], Tatamitani[39], and Yamato [42][43]) Let $n \geq 3$ and suppose Assumption 0 holds. Then, every SCC F satisfying Maskin monotonicity and no veto power can be doubly implemented.*

We show that no veto power is sufficient for partially honest double implementation under Assumption 1.

Theorem 2. *Let $n \geq 3$ and suppose Assumption 1 holds. Then, every SCC F satisfying no veto power can be partially honest doubly implemented.*

The proof of Theorem 2 is given in Appendix

The following property is important for partially honest double implementation under Assumption 2.

Definition 5. An SCC F satisfies *unanimity* if for each $R \in \mathcal{D}$ and each $a \in A$, if for each $i \in N$, $L(R_i, a) = A$, then $a \in F(R)$.

Unanimity says that if an outcome $a \in A$ is at the top of all agents' preference orderings, then the outcome a is F-optimal at the preference profile.

Our main result is that unanimity is sufficient for partially honest double implementation under Assumption 2.

Theorem 3. *Let $n \geq 3$ and suppose Assumption 2 holds. Then, every SCC F satisfying unanimity can be partially honest doubly implemented.*

The proof of Theorem 3 is given in Appendix..

Remark. The strong Pareto correspondence **SP** satisfies unanimity but violates Maskin monotonicity and no veto power for some domain:

Strong Pareto correspondence (SP): $\mathbf{SP}(R) = \{a \in A | \nexists b \in A$ such that for each $i \in N$, bR_ia, and for some $i \in N, bP_ia\}$

The following example represents that the strong Pareto correspondence **SP** violates Maskin monotonicity and no veto power.

Example 2. Consider the following example. There are three agents, $N = \{1, 2, 3\}$, two outcomes, $A = \{a, b\}$, and two possible preference profiles, $\mathcal{D} = \{R, R'\}$. Preferences are given by :

R_1	R_2	R_3		R'_1	R'_2	R'_3
a, b	a, b	a, b		a, b	a, b	a
						b

The strong Pareto correspondence **SP** evaluated at these two states is $\mathbf{SP}(R) = \{a, b\}$, and $\mathbf{SP}(R') = \{a\}$. However, Maskin monotonicity and no veto power imply that we must have $b \in \mathbf{SP}(R')$.■

Previous studies provide a necessary condition for double implementation under Assumption 0 (Maskin [31] and Yamato [43]).

Theorem 4. *(Maskin [31], Yamato [43]) Let $n \geq 3$ and suppose Assumption 0 holds. If an SCC F does not satisfy Maskin monotonicity, it cannot be doubly implemented.*

By Theorem 4, the strong Pareto correspondence **SP** cannot be doubly implemented under Assumption 0. On the other hand, since the strong Pareto correspondence **SP** satisfies unanimity, the strong Pareto correspondence **SP** can be partially honest doubly implemented under Assumption 2.■

We summarize sufficient conditions for partially honest double implementation under Assumption $k \in \{0, 1, 2\}$ in Figure 1 below.

	Assumption 0	→	Assumption 1	→	Assumption 2
Nash Implementation	Maskin [31]	‖	Dutta and Sen [8]	‖	Kimya [21]
	Maskin monotonicity *no veto power*		*no veto power*		*unanimity*
↓					
Double Implementation	Jackson et al.[16] Tatamitani[39] Yamato [42][43]		This paper (Theorem 2)		This paper (Theorem 3)
	Maskin monotonicity *no veto power*		*no veto power*		*unanimity*

Figure 1

Following Dutta and Sen [8], we consider a truth-telling with regard to a preference profile. On the other hand, motivated by some experiments (Gneezy[10], Hurkens and Kartik[13]), Hagiwara et al.[11] consider a truth-telling with regard to an outcome.

Example 3. In contrast to Example 1, Hagiwara et al.[11] construct a mechanism $\Gamma^O = (M, g)$, which is called an outcome mechanism by them. For each $i \in N$, the message space of agent $i \in N$ consists of $M_i = A \times N$. Denote an element of M_i by $m_i = (a^i, k^i)$. The outcome function $g : M \longrightarrow A$ is defined as follows:

Rule 1 : If there is $i \in N$ such that for each $j \neq i$, $m_j = (a, k^j)$, then $g(m) = a$.
Rule 2 : In all other cases, $g(m) = a^{i^*}$, where $i^* = (\Sigma_{i \in N} k^i)(mod\ n) + 1$.

They define a truth-telling correspondence by $T_i^{\Gamma^O}(R) = F(R) \times N$ for each $i \in N$ and each $R \in \mathcal{D}$.■

We give an example to show that in their outcome mechanism, even if two agents are partially honest, there may be Nash equilibrium outcomes with partially honest agents in which agents use weakly dominated messages, and hence the set of undominated Nash equilibrium outcomes with partially honest agents may be a proper subset of the set of Nash equilibrium outcomes with partially honest agents. On the other hand, if we suppose Assumption n holds, we succeed in strategy space reduction with respect to double implementation.[6] Although Assumption n is stronger, their outcome mechanism solves some problems with respect to Maskin's canonical mechanisms, self-relevancy and the assumption of complete information.[7]

[6]For discussions of strategy space reduction for Nash implementation, see Saijo[35] and Lombardi and Yoshihara[24].

[7]Hurwicz[14] imposes self-relevancy on a mechanism: each agent must emit information related only to her own characteristics. In the general environment, a typical self-relevant mechanism is a direct revelation mechanism where each agent announces her own preference only. On the other hand, Tatamitani [40][41] additionally requires each agent to announce an outcome, which could be regarded as self-relevant information in the general environment because preference revelation alone is too restrictive (see Dasgupta et al. [7]). Saijo et al.[36] regard quantity and price-quantity mechanisms as self-relevant mechanism in the exchange economics.

Example 4.[8] Consider the following example. There are three agents, $N = \{1, 2, 3\}$ such that agent 1 is only self-interested, i.e., $H = \{2, 3\}$, three outcomes, $A = \{a, b.c\}$, and two admissible preference profiles, $\mathcal{D} = \{R, R'\}$. Preferences are given by :

R_1	R_2	R_3	R'_1	R'_2	R'_3
a	c	c	a	c	c
b	b	b	b	a	b
c	a	a	c	b	a

Define the SCC F as follows: $F(R) = \{c\}$, and $F(R') = \{a\}$. Note that the SCC F satisfies unanimity, so that the SCC can be partially honest implemented in Nash equilibria by their outcome mechanism under Assumption 2.

There exists a unique Nash equilibrium outcome with partially honest agents in $(\Gamma^O, (T_i^{\Gamma^O})_{i\in N}, \succsim^{R,H})$, c. If $m_i = (c, k^i)$ for each $i \in N$, $m \in NE(\Gamma^O, (T_i^{\Gamma^O})_{i\in N}, \succsim^{R,H})$ and it is easy to see that the message m_2 and m_3 is not weakly dominated by any message in M_2 and M_3 *at* $\succsim_2^{R,H}$ and $\succsim_3^{R,H}$, respectively. On the other hand, it is easy to see that $m_1 = (c, k^1)$ is weakly dominated by $m'_1 = (a, k^1)$ at $\succsim_1^{R,H}$.

On the other hand, suppose agent 1 is partially honest, and if $m_2 = m_3$, then $(m_1, m_{-1}) \succ_1^{R,H} (m'_1, m_{-1})$. In addition, it is easy to see that the message $m_1 = (c, k^1)$ is not weakly dominated by any message in M_1 *at* $\succsim_1^{R,H}$. That is, if we suppose Assumption n holds, $F(R) = NE_A(\Gamma^O, (T_i^{\Gamma^O})_{i\in N}, \succsim^{R,H}) = UNE_A(\Gamma^O, (T_i^{\Gamma^O})_{i\in N}, \succsim^{R,H}) = \{c\}$.■

5 Implications

In this section, we derive a number of corollaries in problems of allocating an infinitely divisible resource, coalitional games, general problems of one-to-one matching, and voting games. Since all SCCs considered here violate Maskin monotonicity, the SCCs cannot be doubly implemented under Assumption 0 by Theorem 4. But the SCCs can be doubly implemented with partially honest agents.

5.1 Problems of Allocating an Infinitely Divisible Resource

In this subsection, we consider a problem of allocating an infinitely divisible resource among a group of agents. A problem of allocating an infinitely divisible resource is a triple $(N, A(M), R)$. The first component $N = \{1, ..., n\}$ with $n \geq 3$ is a set of agents among whom an amount $M \in \mathbb{R}_{++}$ of an infinitely divisible resource has to be allocated. Note that we do not assume that the resource can be disposed of. Given $M \in \mathbb{R}_{++}$, an *allocation for* M is a list $a \in \mathbb{R}_+^N$ such that $\Sigma_{i\in N} a_i = M$. The second component $A(M) = \{a \in \mathbb{R}_+^N | \Sigma_{i\in N} a_i = M\}$ is the set of *allocations.* The third component $R = (R_1, ..., R_n)$ where R_i is a preference ordering for agent $i \in N$ over A is a preference profile. Let P_i and I_i be the asymmetric and symmetric components of R_i, respectively. We assume that preferences are continuous; that is, for each $a_i \in \mathbb{R}_+$, the sets $\{b_i \in \mathbb{R}_+ | b_i R_i a_i\}$ and $\{b_i \in \mathbb{R}_+ | a_i R_i b_i\}$ are closed. Let $\tilde{\Re}_i$ be the set of all continuous preference orderings for

[8]Yamato[43] uses this example in which for Maskin's mechanism, the set of undominated Nash equilibrium outcomes may be strictly smaller than the set of Nash equilibrium outcomes.

agent $i \in N$. Let $\tilde{\Re} = \times_{i \in N} \tilde{\Re}_i$. Let $\mathcal{D} = \times_{i \in N} \mathcal{D}_i \subseteq \tilde{\Re}$ where $\mathcal{D}_i \subseteq \tilde{\Re}_i$ for each $i \in N$ be a domain.

We consider a situation in which the mechanism designer does not know agents' preferences. This situation is modeled by the triple $(N, A, \mathcal{D})$, which we refer to as a *division problem environment of an infinitely divisible resource.*

In addition to a domain $\mathcal{D} = \tilde{\Re}$, we focus on domains satisfying the following restrictions:

Single-plateaued preferences. Given $(R_i, M) \in \tilde{\Re}_i \times \mathbb{R}_{++}$, let $T(R_i, M) = \{a \in [0, M] | aR_i b$ for each $b \in [0, M]\}$ be the top set for (R_i, M). Note that since R_i is continuous, $T(R_i, M) = \phi$ for each $(R_i, M) \in \tilde{\Re}_i \times \mathbb{R}_{++}$. Let $\bar{\mathrm{T}}(R_i, M) = max\ T(R_i, M)$ and $\underline{\mathrm{T}}(R_i, M) = min\ T(R_i, M)$. A preference ordering $R_i \in \tilde{\Re}_i$ is single-plateaued on $[0, M]$ if there is an interval $[\underline{\mathrm{T}}(R_i, M), \bar{T}(R_i, M)] \subseteq [0, M]$ such that for each $a, b \in [0, M]$, if $b < a \leq \underline{\mathrm{T}}(R_i, M)$ or $\bar{\mathrm{T}}(R_i, M) \leq a < b$, then $aP_i b$; if $\underline{\mathrm{T}}(R_i, M) \leq a \leq b \leq \bar{T}(R_i, M)$, then $aI_i b$. Note that $T(R_i, M) = [\underline{\mathrm{T}}(R_i, M), \bar{T}(R_i, M)]$. Let $\mathcal{D}_i^{SPL}$ be the set of single-plateaued preference orderings on $[0, M]$ for agent $i \in N$ and $\mathcal{D}^{SPL} = \times_{i \in N} \mathcal{D}_i^{SPL}$ be the single-plateaued domain on $[0, M]$.

Single-dipped preferences. A preference ordering $R_i \in \tilde{\Re}_i$ is single-dipped on $[0, M]$ if there is a point $d(R_i) \in [0, M]$ such that for each $a, b \in [0, M]$, if $a < b \leq d(R_i)$ or $d(R_i) \leq b < a$, then $aP_i b$. Let $\mathcal{D}_i^{SD}$ be the set of single-dipped preference orderings on $[0, M]$ for agent $i \in N$ and $\mathcal{D}^{SD} = \times_{i \in N} \mathcal{D}_i^{SD}$ be the single-dipped domain on $[0, M]$.

Let us give an example of an SCC.

Strong Pareto correspondence (SP): SP$(R) = \{a \in A(M) | \nexists b \in A(M)$ such that for each $i \in N$, $b_i R_i a_i$, and for some $i \in N, b_i P_i a_i\}$

If $\mathcal{D} = \Re$ or $\mathcal{D}^{SPL}$, it is well-known that the strong Pareto correspondence **SP** violates Maskin monotonicity. If $\mathcal{D} = \mathcal{D}^{SD}$, Inoue and Yamamura[15] show that any selection from the strong Pareto correspondence **SP** does not satisfy Maskin monotonicity. By Theorem 4, if $\mathcal{D} = \Re$, $\mathcal{D}^{SPL}$, or $\mathcal{D}^{SD}$, the strong Pareto correspondence **SP** cannot be doubly implemented under Assumption 0.

On the other hand, it is well-known that the strong Pareto correspondence **SP** satisfies unanimity but violates no veto power. We conclude that by changing Assumption 0 into Assumption 2, the strong Pareto correspondence **SP** can be partially honest doubly implemented.

Corollary 1. *Let* $n \geq 3$ and $\mathcal{D} = \Re$, $\mathcal{D}^{SPL}$, or $\mathcal{D}^{SD}$. *Suppose Assumption 2 holds. Let* $(N, A(M), \mathcal{D})$ *be a division problem environment of an infinitely divisible resource. Then, the strong Pareto correspondence* **SP** *can be partially honest doubly implemented.*

5.2 Coalitional Games

In this subsection, we consider a coalitional game. A coalitional game (N, A, R, v) contains a finite set of agents N with $n \geq 3$, a non-empty set of outcomes A, a preference profile $R \in \mathcal{D}$, and a characteristic function $v : 2^N \backslash \{\phi\} \to 2^A$, which assigns for each coalition $S \in 2^N \backslash \{\phi\}$ a subset of outcomes. Given a coalitional game (N, A, R, v), an outcome

$a \in A$ is *weakly blocked by* S if there is $b \in v(S)$ such that bR_ia for each $i \in S$, and bP_ia for some $i \in S$.

We consider a situation in which the mechanism designer knows what is feasible for each coalition, that is, the characteristic function v, but she does not know agents' preferences. This situation is modeled by the four-tuple $(N, A, \mathcal{D}, v)$, which we refer to as a *coalitional game environment*.

Let us give an example of an SCC.

Strong core correspondence(SC): $\mathbf{SC}(R) = \{a \in v(N)$ — a is not weakly blocked by any coalition $S\}$

We say that $(N, A, \mathcal{D}, v)$ is *a coalitional game environment with non-empty strong core* if $\mathbf{SC}(R) \neq \phi$ for each $R \in \mathcal{D}$.

Lombardi and Yoshihara[26] show that the strong core correspondence **SC** does not satisfy Maskin monotonicity.[9] By Theorem 4, the strong core correspondence **SC** cannot be doubly implemented under Assumption 0.

On the other hand, it is well-known that the strong core correspondence **SC** satisfies unanimity but violates no veto power. We conclude that by changing Assumption 0 into Assumption 2, **SC** can be partially honest doubly implemented.

Corollary 2. *Let $n \geq 3$ and suppose Assumption 2 holds. Let $(N, A, \mathcal{D}, v)$ be a coalitional game environment with non-empty strong core. Then, the strong core correspondence* ***SC*** *can be partially honest doubly implemented.*

5.3 General Problems of one-to-one Matching

In this subsection, we consider a general problem of one-to-one matching (Sönmez [38], Ehlers [9]). A *generalized matching problem* is a triple (N, S, R). The first component N is a finite set of agents with $n \geq 3$. The second component $S = (S_i)_{i \in N}$ is a profile of subsets of N with $i \in S_i$ for each $i \in N$. Here, S_i represents the set of possible assignments for agent i. The last component $R = (R_1, ..., R_n)$ where R_i is a preference ordering for agent $i \in N$ over S_i is a preference profile. Let P_i and I_i be the asymmetric and symmetric components of R_i, respectively. Let $\Re_i$ be the set of all preference orderings for agent $i \in N$ and $\Re = \times_{i \in N} \Re_i$ be the set of all preference profiles. Given $i \in N$, let $\tilde{\Re}_i$ denote the set of all preference orderings for agent i under which agent i is indifferent between at most two distinct assignments. Let $\tilde{\Re} = \times_{i \in N} \tilde{\Re}_i$. Throughout the paper, we fix a domain $\mathcal{D} = \times_{i \in N} \mathcal{D}_i$ where $\mathcal{D}_i$ for each $i \in N$ such that $\tilde{\Re} \subseteq \mathcal{D} \subseteq \Re$.

A *matching* is a bijection $\mu : N \to N$ such that each agent's assignment belongs to his set of possible assignments, i.e., for each $i \in N$, $\mu(i) \in S_i$. Given $T \subseteq N$, let $\mu(T)$ denote the *set of assignments of the agents in T at μ*, i.e., $\mu(T) = \{\mu(i) | i \in T\}$. Let $\mathcal{M}$ denote the *set of all matchings*. Let μ^I denote the matching such that for each $i \in N$, $\mu(i) = i$. We specify a subset $\mathcal{M}^f$ of $\mathcal{M}$ as the *set of feasible matchings*. We always require that $\mu^I \in \mathcal{M}^f$ and for each $i \in N$, $S_i = \{\mu(i) | \mu \in \mathcal{M}^f\}$. In the context of matching problems, the set of allocations A is the set of feasible matchings $\mathcal{M}^f$.

[9] Moreover, Lombardia and Yoshihara[26] show that the strong core correspondence can not be partially honest implemented in Nash equilibria under Assumption 1.

We consider a situation in which the mechanism designer does not know agents' preferences. This situation is modeled by the triple $(N, \mathcal{M}^f, \mathcal{D})$, which we refer to as a *generalized matching problem environment*.

Since N, S, and $\mathcal{M}^f$ remain fixed, a generalized matching problem is simply a preference profile $R \in \mathcal{D}$. Given a preference ordering R_i of an agent $i \in N$, initially defined over S_i, we extend it to the set of feasible matchings $\mathcal{M}^f$ in the following natural way: agent i prefers the matching μ to the matching μ' if and only if he prefers his assignment under μ to his assignment under μ'. Slightly abusing notation, we use the same symbols to denote preferences over possible assignments and preferences over feasible matchings.

An SCC is a mapping $F : \mathcal{D} \twoheadrightarrow \mathcal{M}^f$ that specifies a non-empty subset $F(R) \subseteq \mathcal{M}^f$ for each $R \in \mathcal{D}$. For each $R \in \mathcal{D}$, each matching $\mu \in F(R)$ is interpreted to be desirable when the preference profile is $R \in \mathcal{D}$. An SCC F is *non-empty* if for each $R \in \mathcal{D}$, $F(R) \neq \phi$. We allow solutions to be empty.

We consider situations in which only certain coalitions can coordinate their actions. A coalition structure is a set of coalitions. In particular, each agent "voluntarily" participates in the matching. Formally, a *coalition structure* is a set $\mathcal{T} \subseteq 2^N \backslash \{\phi\}$ such that for each $i \in N$, $\{i\} \in \mathcal{T}$. Given $R \in \mathcal{D}$, $T \in \mathcal{T}$, and $\mu \in \mathcal{M}^f$, we say that *coalition T blocks μ under R* if for some $\tilde{\mu} \in \mathcal{M}^f$, (1) $\tilde{\mu}(T) = T$, (2) for each $i \in T$, $\tilde{\mu}(i) R_i \mu(i)$, and (3) for some $j \in T$, $\tilde{\mu}(i) P_i \mu(i)$.

Let us give an example of an SCC.

Strong $\mathcal{T}$-core correspondence ($\mathbf{SC}^{\mathcal{T}}$): $\mathbf{SC}^{\mathcal{T}}(R) = \{\mu \in \mathcal{M}^f \mid$ there is no $T \in \mathcal{T}$ that blocks μ under $R\}$.

The strong $\mathcal{T}$-core correspondence chooses the feasible matchings that are not blocked by any coalition in $\mathcal{T}$ for each $R \in \mathcal{D}$. Throughout this paper, we assume that N, S, and $\mathcal{M}^f$ are such that the strong $\mathcal{T}$-core is non-empty for each preference profile.

Ehlers [9] shows that the strong $\mathcal{T}$-core correspondence $\mathbf{SC}^{\mathcal{T}}$ does not satisfy Maskin monotonicity. By Theorem 4, the strong $\mathcal{T}$-core correspondence $\mathbf{SC}^{\mathcal{T}}$ cannot be doubly implemented under Assumption 0.

On the other hand, it is well-known that the strong $\mathcal{T}$-core correspondence $\mathbf{SC}^{\mathcal{T}}$ satisfies unanimity but violates no veto power. We conclude that by changing Assumption 0 into Assumption 2, the strong $\mathcal{T}$-core correspondence $\mathbf{SC}^{\mathcal{T}}$ can be partially honest doubly implemented.

Corollary 3. *Let $n \geq 3$ and suppose Assumption 2 holds. Let $(N, \mathcal{M}^f, \mathcal{D})$ be a generalized matching problem environment. Then, strong $\mathcal{T}$-core correspondence $\mathbf{SC}^{\mathcal{T}}$ can be partially honest doubly implemented.*

5.4 Voting Games

In this subsection, we consider a voting game. A *voting game* (N, A, R) contains a finite set of agents N with $n \geq 3$, a non-empty finite set of outcomes A, and a preference profile $R \in \mathcal{D}$.

We consider a situation in which the mechanism designer does not know agents' preferences. This situation is modeled by the triple $(N, A, \mathcal{D})$, which we refer to as a *voting game environment*.

For each $R \in \mathcal{D}$ and each $a, b \in A$, we write $aD(R)b$ if a strict majority of agents prefer a to b. For each $R \in \mathcal{D}$, let $B^i(a, R) = k$ if $a \in A$ is the k'th most preferred outcome.

We consider the following interesting SCCs, the top-cycle correspondence and the Borda correspondence.

Top-cycle correspondence (tc): $\mathbf{tc}(R) = \cap\{B \subseteq A — a \in B, b \notin B \text{ implies } aD(R)b\}$.

The top-cycle correspondence **tc** at $R \in \mathcal{D}$ is the smallest subset of A with the property that nothing outside the set is preferred by a strict majority to anything in the set.

Palfley and Srivastava [**?**] show that the top-cycle correspondence **tc** does not satisfy Maskin monotonicity. By Theorem 4, the top-cycle correspondence **tc** cannot be doubly implemented under Assumption 0.

On the other hand, it is well-known that the top-cycle correspondence **tc** satisfies no veto power. We conclude that by changing Assumption 0 into Assumption 2, top-cycle correspondence **tc** can be partially honest doubly implemented.

Corollary 4. *Let $n \geq 3$ and suppose Assumption 1 holds. Let $(N, A, \mathcal{D})$ be a voting game environment. Then,* the top-cycle correspondence **tc** *can be partially honest doubly implemented.*

Borda correspondence ($\mathbf{F}_B$): $\mathbf{F}_B(R) = \{a \in A | \Sigma_{i \in N} B^i(a, R) \leq \Sigma_{i \in N} B^i(b, R)$ for each $b \in A\}$.

The following example represents that the Borda correspondence violates Maskin monotonicity and no veto power.

Example 5. Consider the following example. There are three agents, $N = \{1, 2, 3\}$, two outcomes, $A = \{a, b, c\}$, and two possible preference profiles, $\mathcal{D} = \{R, R'\}$. Preferences are given by :

R_1	R_2	R_3	R'_1	R'_2	R'_3
a	b	c	a	a, b, c	c
b	c	b	b, c		b
c	a	a			a

The Borda correspondence $\mathbf{F}_B$ evaluated at these two preference profiles is $\mathbf{F}_B(R) = \{b\}$, and $\mathbf{F}_B(R') = \{c\}$. However, Maskin monotonicity and no veto power imply that we must have $b \in \mathbf{F}_B(R')$ and $a \in \mathbf{F}_B(R')$, respectively.■

By Theorem 4, the Borda correspondence $\mathbf{F}_B$ cannot be doubly implemented under Assumption 0.

On the other hand, it is well-known that the Borda correspondence $\mathbf{F}_B$ satisfies unanimity. We conclude that by changing Assumption 0 into Assumption 2, the Borda correspondence $\mathbf{F}_B$ can be partially honest doubly implemented.

Corollary 5. *Let $n \geq 3$ and suppose Assumption 2 holds. Let $(N, A, \mathcal{D})$ be a voting game environment. Then, the Borda correspondence F_B can be partially honest doubly implemented.*

6 Concluding Remarks

In this paper, we are concerned with the design of a mechanism that doubly implements an SCC with partially honest agents. We show that if there are at least three agents and at least one agent is partially honest, no veto power is sufficient for double implementation with partially honest agents (Theorem 2). Therefore, we no longer need Maskin monotonicity as a necessary condition of double implementability. Moreover, we show that if there are at least three agents and at least two agents are partially honest, unanimity is sufficient for double implementation with partially honest agents (Theorem 3). Hence, more social choice correspondences can be doubly implemented with partially honest agents if at least two agents are partially honest since unanimity is weaker than no veto power.

Our results provide positive implications for some problems under Assumption 1 or Assumption 2. However, our results are silent with respect to double implementability of SCCs that do not satisfy no veto power under Assumption 1. Therefore, as Lombardi and Yoshihara[26] provide, we need to study a full characterization of doubly implementable social choice correspondences under Assumption 1 and investigate whether the SCCs can be doubly implemented under Assumption 1.

7 Appendix

Proof of Theorem 2: Let F be an SCC satisfying no veto power. We construct a mechanism $\Gamma = (M, g)$. For each $i \in N$, the message space of agent $i \in N$ consists of $M_i = \mathcal{D} \times A \times A \times \{-n, ..., -1, 0, 1, ..., n\}$. Denote an element of M_i by $m_i = (R^i, a^i, b^i, k^i)$. For each $i \in N$ and each $R_i \in \mathcal{D}_i$, define $\bar{\text{b}}(R_i)$ and $\underline{\text{b}}(R_i)$ as follows: (1) if there exist $b, c \in A$ such that bP_ic, then let $\bar{\text{b}}(R_i) = b$ and $\underline{\text{b}}(R_i) = c$; (2) otherwise, pick any $b, c \in A$ with $b \neq c$, let $\bar{\text{b}}(R_i) = b$ and $\underline{\text{b}}(R_i) = c$. The outcome function $g : M \longrightarrow A$ is defined as follows:

Rule 1 : If for some $i \in N$, $m_j = (R, a, \cdot, j)$ such that $a \in F(R)$ for each $j \neq i$, then

$$g(m) = a.$$

Rule 2 : If for some $i \in N$, $m_j = (R, a, \cdot, -i)$ such that $a \in F(R)$ for each $j \neq i$, then

$$g(m) = \begin{cases} \bar{\text{b}}(R_i) \text{ if } m_i = (R, a, \bar{b}(R_i), i) \\ \underline{\text{b}}(R_i) \text{ if } m_i \neq (R, a, \bar{b}(R_i), i) \text{ with } k^i \leq 0 \text{ or } k^i = i. \end{cases}$$

Rule 3 : In all other cases, $g(m) = a^{i^*}$, where $i^* = (\Sigma_{i \in N} \max\{0, k^i\})(mod\ n) + 1$.

For each $i \in N$ and each $R \in \mathcal{D}$, a truth-telling correspondence is defined by $T_i^\Gamma(R, F) = \{R\} \times A \times A \times \{-n, ..., -1, 0, 1, ..., n\}$.

The proof consists of three lemmata.

Lemma 1. Let $R \in \mathcal{D}$, $H \in \mathcal{H}^1$, and $a \in F(R)$ be given. If for each $i \in N$, $m_i = (R, a, \bar{\text{b}}(R_i), i)$, then $m \in NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.

Proof: For each $i \in N$, let $m_i = (R, a, \bar{\text{b}}(R_i), i)$. By Rule 1, $g(m) = a$. No unilateral deviation can change the outcome and $m_i \in T_i^\Gamma(R)$ for each $i \in N$. Hence,

$m \in NE(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H})$.■

Lemma 2. Let $R \in \mathcal{D}$, $H \in \mathcal{H}^1$, and $a \in F(R)$ be given. If for each $i \in N$, $m_i = (R, a, \bar{b}(R_i), i)$ is undominated at $\succsim_i^{R,H}$.

Proof: First, suppose that there exist $b, c \in A$ with bP_ic. Then, $\bar{\text{b}}(R_i)P_i\underline{\text{b}}(R_i)$. We show that for each $\tilde{m}_i \neq m_i$, there exists $\tilde{m}_{-i} \in M_{-i}$ such that $(m_i, \tilde{m}_{-i}) \succ_i^{R,H} (\tilde{m}_i, \tilde{m}_{-i})$. There are two cases to consider.

case 1. $\tilde{k}^i \leq 0$ or $\tilde{k}^i = i$.

Let $\tilde{m}_j = (R, a, \cdot, -i)$ for each $j \neq i$. By Rule 2, $g(m_i, \tilde{m}_{-i}) =\bar{\text{b}}(R_i)$ and $g(\tilde{m}_i, \tilde{m}_{-i}) =\underline{\text{b}}(R_i)$, so that $(m_i, \tilde{m}_{-i}) \succ_i^{R,H} (\tilde{m}_i, \tilde{m}_{-i})$.

case 2. $\tilde{k}^i > 0$ and $\tilde{k}^i \neq i$.

Define $\tilde{m}_{-i} \in M_{-i}$ as follows: for some $j \neq i$, $\tilde{m}_j = (R', a', \underline{\text{b}}(R_i), j-1)$, for some $h \neq i, j$, $\tilde{m}_h = (R'', a'', \underline{\text{b}}(R_i), \tilde{k}^h)$, and for any other ℓ, $\tilde{m}_\ell = (\cdot, \cdot, \underline{\text{b}}(R_\ell), \tilde{k}^\ell)$, where $(R, a) \neq (R', a') \neq (R'', a'')$ and $(\Sigma_{h\neq i,j}\tilde{k}^h + i + j - 1)(mod\ n) + 1 = i$ with $\tilde{k}^h \geq 0$ for $h \neq i, j$. By Rule 3, $g(m_i, \tilde{m}_{-i}) =\bar{\text{b}}(R_i)$ and $g(\tilde{m}_i, \tilde{m}_{-i}) =\underline{\text{b}}(R_i)$, so that $(m_i, \tilde{m}_{-i}) \succ_i^{R,H} (\tilde{m}_i, \tilde{m}_{-i})$.

Next, suppose that for each $b, c \in A$, bI_ic. Obviously, m_i is undominated *at* $\succsim_i^{R,H}$.■

Lemma 3. For each $R \in \mathcal{D}$ and each $H \in \mathcal{H}^1$, $NE_A(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H}) \subseteq F(R)$.

Proof: There are two cases to consider.

Case 1. For each $i \in N$, $m_i = (R', a, \cdot, i)$ such that $R' \neq R$ and $a \in F(R')$.

We show that if $g(m) \notin F(R)$, then $m \notin NE(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H})$. Under Assumption 2, there exists a partially honest agent $h \in H$. Let $m'_h = (R, a'^h, b'^h, k'^h)$. By the definition of the truth-telling correspondence, $m_h \notin T_h^\Gamma(R)$ and $m'_h \in T_h^\Gamma(R)$. By Rule 1, $g(m'_h, m_{-h}) = a$ so that $g(m'_h, m_{-h}) = g(m)$. Since $h \in H$, $(m'_h, m_{-h}) \succ_h^{R,H} (m_h, m_{-h})$. Hence, $m \notin NE(\Gamma, (T_i^\Gamma)_{i\in N}, \succsim^{R,H})$.

Case 2. There are $i, j \in N$ $(i \neq j)$ such that $R^i \neq R^j$.

Let the outcome be some $b \in A$. Then, any one of $(n-1)$ agents can deviate, precipitate the modulo game, and be the winner of the modulo game. Clearly, if the original announcement is to be a Nash equilibrium with partially honest agents, then it must be the case that $L(R_i, b) = A$ for $(n-1)$ agents. Then since F satisfies no veto power, $b \in F(R)$.■

Proof of Theorem 3: Let F be an SCC satisfying unanimity. We construct a mechanism $\Gamma = (M, g)$. For each $i \in N$, the message space of agent $i \in N$ consists of $M_i = \mathcal{D} \times A \times A \times \{-n, ..., -1, 0, 1, ..., n\}$. Denote an element of M_i by $m_i = (R^i, a^i, b^i, k^i)$. The outcome function $g : M \longrightarrow A$ is defined as follows:

Rule 1 : If for some $i \in N$, $m_j = (R, a, \cdot, j)$ such that $a \in F(R)$ for each $j \neq i$, then

$$g(m) = a.$$

Rule 2 : If for some $i \in N$, $m_j = (R, a, \cdot, -i)$ such that $a \in F(R)$ for each $j \neq i$, then

$$g(m) = \begin{cases} \bar{\text{b}}(R_i) \text{ if } m_i = (R, a, \bar{b}(R_i), i) \\ \underline{\text{b}}(R_i) \text{ if } m_i \neq (R, a, \bar{b}(R_i), i) \text{ with } k^i \leq 0 \text{ or } k^i = i. \end{cases}$$

Rule 3 : In all other cases, $g(m) = a^{i^*}$, where $i^* = (\Sigma_{i \in N} \max\{0, k^i\})(mod\ n) + 1$.

For each $i \in N$ and each $R \in \mathcal{D}$, a truth-telling correspondence is defined by $T_i^\Gamma(R) = \{R\} \times A \times A \times \{-n, ..., -1, 0, 1, ..., n\}$.

The proof consists of three lemmata.

Lemma 4. Let $R \in \mathcal{D}$, $H \in \mathcal{H}^2$, and $a \in F(R)$ be given. If for each $i \in N$, $m_i = (R, a, \bar{\mathrm{b}}(R_i), i)$, then $m \in NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.

Lemma 5. Let $R \in \mathcal{D}$, $H \in \mathcal{H}^2$, and $a \in F(R)$ be given. If for each $i \in N$, $m_i = (R, a, \bar{b}(R_i), i)$ is undominated *at* $\succsim_i^{R,H}$.

The proof of Lemma 4 and Lemma 5 are omitted. It follows from the same reasoning as Lemma 1 and Lemma 2, respectively.

Lemma 6. For each $R \in \mathcal{D}$ and each $H \in \mathcal{H}^2$, $NE_A(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H}) \subseteq F(R)$.

Proof: We show that if $g(m) \notin F(R)$, then $m \notin NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$. There are four cases to consider.

Case 1. For each $i \in N$, $m_i = (R', a, \cdot, i)$ such that $R' \neq R$ and $a \in F(R')$.

By the same argument as Case 1 of Lemma 3, $m \notin NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.

Case 2. There is $i \in N$ such that for each $j \neq i$, $m_j = (R', a, \cdot, j)$ such that $R' \neq R$ and $a \in F(R')$, and $m_i \neq (R', a, \cdot, i)$.

By Rule 1, $g(m) = a \in F(R')$ such that $R' \neq R$. Under Assumption 2, since $|H| \geq 2$ there exists a partially honest agent $h \in H \backslash \{i\}$.[10] Without loss of generality, let $i = 1$ and $h = 2$. Let $m_2' = (R, a'^2, b'^2, k'^2)$ be such that $(\Sigma_{j \neq 2} k^j + k'^2)(mod\ n) + 1 = 3$. By the definition of the truth-telling correspondence, $m_2 \notin T_2^\Gamma(R)$ and $m_2' \in T_2^\Gamma(R)$. By Rule 3, $g(m_2', m_{-2}) = a^3 = a$ so that $g(m_2', m_{-2}) = g(m)$. Since agent 2 is partially honest, $(m_2', m_{-2}) \succ_2^{R,H} (m_2, m_{-2})$. Hence, $m \notin NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.

Case 3. Rule 2 is applied.

Suppose $g(m) \notin F(R)$. Since F satisfies unanimity, there is $\ell \in N$ and $b \in A$ such that $bP_\ell g(m)$. Suppose $\ell = i$. Let $m_i' = (\cdot, \cdot, b, i-1)$ if $i \neq 1$ and $m_i' = (\cdot, \cdot, b, n)$ if $i = 1$. By Rule 3, $g(m_i', m_{-i}) = b$ so that $g(m_i', m_{-i}) P_i g(m)$. Otherwise (i.e. $\ell \neq i$), if agent ℓ deviate to $m_\ell' = (\cdot, \cdot, b, k'^\ell) \neq m_\ell$ such that $(\Sigma_{j \neq \ell} k^j + k'^\ell)(mod\ n) + 1 = \ell$, then by Rule 3, $g(m_\ell', m_{-\ell}) = b$ so that $g(m_\ell', m_{-\ell}) P_\ell g(m)$. Whether agent ℓ is partially honest or not, $(m_\ell', m_{-\ell}) \succ_\ell^{R,H} (m_\ell, m_{-\ell})$. Hence, $m \notin NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.

Case 4. Rule 3 is applied.

Suppose $g(m) \notin F(R)$. Since F satisfies unanimity, there is $i \in N$ and $b \in A$ such that $bP_i g(m)$. Let $m_i' = (\cdot, \cdot, b, k'^i) \neq m_i$ be such that $(\Sigma_{j \neq i} k^j + k'^i)(mod\ n) + 1 = i$. By Rule 3, $g(m_i', m_{-i}) = b$ so that $g(m_i', m_{-i}) P_i g(m)$. Whether agent i is partially honest or not, $(m_i', m_{-i}) \succ_i^{R,H} (m_i, m_{-i})$. Hence, $m \notin NE(\Gamma, (T_i^\Gamma)_{i \in N}, \succsim^{R,H})$.■

[10]Note that under Assumption 1, there is no partially honest agent in $N \backslash \{i\}$ when $|H| = 1$ and agent i is partially honest.

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