

# Asymptotic Distributions of Several Test Statistics for High-Dimensional Mean Vector

Shota Katayama\*, Yutaka Kano and Muni S. Srivastava

Osaka University, Japan. *sfujimoto@sigmath.es.osaka-u.ac.jp*  
Osaka University, Japan.  
University of Toronto, Canada.

**Abstract:** The problem of testing hypotheses concerning the mean vector for high-dimensional data has been investigated by many authors. They have proposed several test statistics and obtained their asymptotic distributions, under somewhat restrictive conditions, when both the sample size and the dimension tend to infinity. Indeed, their conditions exclude a typical situation where a population covariance matrix has spiked eigenvalues. For instance, the population covariance matrix with the compound symmetry structure (the variances are the same; the covariances are the same) does not meet their conditions. There are two contributions in this paper. First, we relax their conditions to include such important cases. It is found that the type of asymptotic distributions of their statistics depends on the population covariance matrix. A rather non-standard asymptotic distribution is obtained which is the convolution of normal and chi-squared distributions for a certain population covariance matrix. Secondly, we propose a class of test statistics. It is shown that the statistics in the class have the asymptotic normality with no conditions for the population covariance matrix. In addition, we obtain an optimal test statistic which has the largest asymptotic power in the class.