An Improved Two-Stage Optimization-Based Framework for Unequal-Areas Facility Layout

Miguel F. Anjos

Professor and Canada Research Chair
INRIA International Chair
Academic Director, Institut de l'énergie Trottier

POLYTECHNIQUE MONTRÉAL

INSTITUT DE L'ÉNERGIE TROTTIER

GERAD
GROUP FOR RESEARCH IN DECISION ANALYSIS

Workshop on Advances in Optimization – Tokyo, Japan – August 12, 2016

Co-Author and Reference

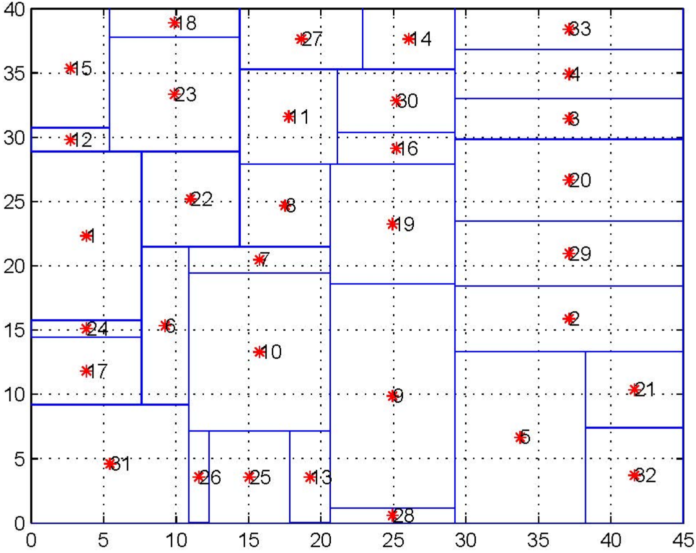
This is joint work with Manuel Vieira (University Nova of Lisbon).

M.F. Anjos and M.V.C. Vieira.
An improved two-stage optimization-based framework for unequal-areas facility layout.
Optimization Letters, first online: 03 February 2016.
DOI: 10.1007/s11590-016-1008-6

The Unequal-Areas Facility Layout Problem

- Find the optimal positions for a given collection of rectangular departments of fixed area within a rectangular facility of fixed area.
- All the dimensions may be given or left undetermined.
- The objective is to minimize (according to a given norm) the distances between pairs of departments that have a nonzero connection cost.
- The problem was originally formulated by Armour & Buffa (1963).
- Like many optimization problems coming from practical applications, facility layout is hard (NP-hard).
- In Very Large Scale Integration (VLSI) design, a very similar problem is referred to as *floorplanning*, and the departments are called modules.

Example of a Layout for a 33-Department Instance



A Well-Studied Problem - Global Optimal Solutions

- Montreuil (1990) proposed one of the first MIP formulations in the continuous plane, where integer variables are used to prevent overlap.
- Castillo and Westerlund (2005) used a MIP formulation that satisfies the area requirements within a given accuracy ϵ.
- Major contributions by Meller, Sherali and co-authors.
- Meller et al. (2007) solved instances with up to 11 departments; the previous limit was nine departments by Sherali et al. (2003).
- Xie & Sahinidis (2008) use a minimum-cost network flow problem to obtain a feasible layout from the sequence-pair representation of the relative position layout. This is only valid for a restricted version of layout.

A Well-Studied Problem - Two-Stage Approaches

These approaches date back at least to the DISCON method of Drezner (1980).

- Solve the first stage model that provides the relative position of the departments.
- The solution to the first stage is used as the starting point for the second stage.
- The second stage computes a feasible layout.

Recent Two-Stage Approaches

A. and Vannelli (2002,2006) use a first stage that approximates the departments using circles of radius r_i centered at (x_i, y_i):

$$\begin{array}{rl}\min\limits_{(x_i,y_j),w_F,h_F} & \sum\limits_{1\le i<j\le n} c_{ij}D_{ij} + \left(\frac{t_{ij}}{D_{ij}} - 1\right) \\ \text{s.t.} & x_i + r_i \le \frac{1}{2}w_F \text{ and } r_i - x_i \le \frac{1}{2}w_F, \text{ for } i = 1, ..., N, \\ & y_i + r_i \le \frac{1}{2}h_F \text{ and } r_i - y_i \le \frac{1}{2}h_F, \text{ for } i = 1, ..., N,\end{array}$$

where $D_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2$,
and $t_{ij} = \alpha(r_i + r_j)^2$ is a *target distance* between i and j ($\alpha > 0$ fixed).

The interpretation is:

- The first term is an attractor (aims for $D_{ij} = 0$).
- The second term is a repeller that prevents the circles from overlapping.

Recent Two-Stage Approaches (ctd)

Jankovits, Luo, A. and Vannelli (2011) proposed an improvement of the A.-Vannelli framework:

- An improved first stage model that incorporates the aspect ratio constraints in a limited manner.
- A new formulation of the second stage as a (convex) second-order conic optimization problem.
- This approach provided the best results for the Armour & Buffa 20-department instance (AB20) until now.
- Instances with up to 30 departments can be solved in up to 5 minutes.

Recent Two-Stage Approaches (ctd)

Kulturel-Konak & Konak (2013) proposed the following approach:

- The GA searches for the relative locations of the departments.
- The LP model determines their exact locations and shapes.
- They introduce the location/shape representation: department i is represented as (x_i, y_i, α_i), where $\alpha_i = h_i/w_i$.
- For each (x_i, y_i, α_i), define two straight lines:
  - One passing through (x_i, y_i) and the upper right corner $(x_i + w_i/2, y_i + h_i/2)$;
  - the other passing through (x_i, y_i) and the upper left corner $(x_i - w_i/2, y_i + h_i/2)$.
  - These lines split the facility into four regions w.r.t. department i, so every other department is above, below, left or right of i.
- It outperforms the previous approaches: the cost function is reduced and the computational time is lower.

Resende and Gonçalves (2015): Very recent paper that applies a random-key GA followed by an LP.

Our Contribution

We propose an improved version of the framework of Jankovits et al.

- More precise first stage:
  - The departments are handled as rectangles.
  - Aspect ratio constraints are enforced exactly in the first stage.
  - Simpler objective function $\rightarrow$ improved computational times.
- Use the same second stage (second-order conic optimization problem) as Jankovits et al.
- The resulting framework normally improves on the results of Jankovits et al.
- We can compute solutions for instances with up to 100 departments in minutes.

Starting Formulation

$$\min_{x_i, y_i, h_i, w_i} \sum_{1 \le i < j \le N} c_{ij} \left(|x_i - x_j| + |y_i - y_j| \right)$$

$$\text{s.t.} \quad x_i + \frac{1}{2} w_i \le \frac{1}{2} w_F, \quad x_i - \frac{1}{2} w_i \ge -\frac{1}{2} w_F, \quad i = 1, \dots, N,$$

$$y_i + \frac{1}{2} h_i \le \frac{1}{2} h_F, \quad y_i - \frac{1}{2} h_i \ge -\frac{1}{2} h_F, \quad i = 1, \dots, N,$$

$$w_i h_i = a_i, \quad i = 1, \dots, N,$$

$$\max \left\{ \frac{w_i}{h_i}, \frac{h_i}{w_i} \right\} \le \beta, \quad i = 1, \dots, N,$$

$$w_i^{\min} \le w_i \le w_i^{\max} \text{ and } h_i^{\min} \le h_i \le h_i^{\max}, \quad i = 1, \dots, N,$$

$$|x_i - x_j| \ge \frac{1}{2}(w_i + w_j) \text{ or } |y_i - y_j| \ge \frac{1}{2}(h_i + h_j), \quad 1 \le i < j \le N.$$

Objective function

The objective function

$$\sum_{1 \le i < j \le N} c_{ij} \left(|x_i - x_j| + |x_j - x_i| \right)$$

can be linearized term-by-term using the standard technique:

$$\begin{aligned}
\min_{x_i, y_i, u_i, v_i} \quad & \sum_{1 \le i < j \le N} c_{ij}(u_{ij} + v_{ij}), \\
\text{s.t.} \quad & u_{ij} \ge x_j - x_i, \\
& u_{ij} \ge x_i - x_j, \quad 1 \le i < j \le N, \\
& v_{ij} \ge y_j - y_i, \\
& v_{ij} \ge y_i - y_j, \quad 1 \le i < j \le N.
\end{aligned}$$

Area constraints

The area constraint

$$w_i h_i = a_i$$

is not linear nor convex.

It can be relaxed to the following convex version:

$$w_i h_i \geq a_i$$

that is easy to represent as a semidefinite constraint:

$$\begin{pmatrix} w_i & \sqrt{a_i} \\ \sqrt{a_i} & h_i \end{pmatrix} \succeq 0.$$

Any 2×2 semidefinite constraint is equivalent to a second-order cone constraint (see e.g. Kim & Kojima (2001)).

Aspect ratio constraints

The aspect ratios must satisfy

$$\frac{w_i}{h_i} \leq \beta \quad \text{and} \quad \frac{h_i}{w_i} \leq \beta.$$

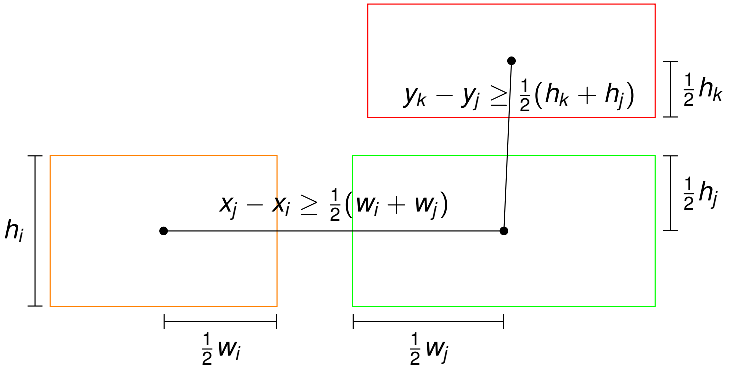
Therefore, we obtain linear constraints

$$w_i - \beta h_i \leq 0 \quad \text{and} \quad h_i - \beta w_i \leq 0.$$

(If $\beta = 1$, then the department is a square.)

No-Overlap Constraints

$$|x_i - x_j| \geq \frac{1}{2}(w_i + w_j) \quad \text{or} \quad |y_i - y_j| \geq \frac{1}{2}(h_i + h_j)$$



First Stage Relaxation of the No-Overlap Constraints

- Drop the no-overlap constraints.
- Define the target distance

$$T_{ij}^2 = \frac{1}{4}\left((w_i + w_j)^2 + (h_i + h_j)^2\right).$$

- The resulting objective function is:

$$\sum_{i,j} c_{ij} D_{ij}^2 + \alpha K \left\{ \frac{T_{ij}^2}{D_{ij}^2} - 1 \right\},$$

where $0 < \alpha \leq 1$ is the parameter to balance the penalty term versus the cost term.

First Stage Model

$$
\begin{aligned}
\min \quad & \sum_{1 \le i < j \le N} c_{ij} D_{ij}^2 + \alpha K \left\{ \frac{T_{ij}^2}{D_{ij}^2} - 1 \right\}, \\
\text{s.t.} \quad & x_i + \frac{1}{2} w_i \le \frac{1}{2} w_F, \quad \text{and} \quad x_i - \frac{1}{2} w_i \ge -\frac{1}{2} w_F, \ \text{ for } i = 1, \dots, N, \\
& y_i + \frac{1}{2} h_i \le \frac{1}{2} h_F, \quad \text{and} \quad y_i - \frac{1}{2} h_i \ge -\frac{1}{2} h_F, \ \text{ for } i = 1, \dots, N, \\
& w_i h_i \ge a_i, \quad \text{for } i = 1, \dots, N, \\
& \beta w_i - h_i \ge 0, \quad \text{for } i = 1, \dots, N, \\
& \beta h_i - w_i \ge 0, \quad \text{for } i = 1, \dots, N, \\
& w_i, h_i \ge 0, \quad \text{for } i = 1, \dots, N.
\end{aligned}
$$

First Stage Solutions

Examples of solutions for the AB20 instance:

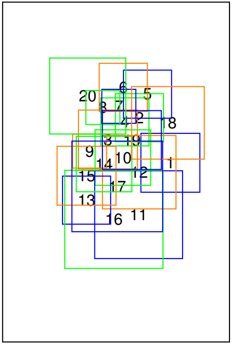


Figure: $\alpha = 0.01$

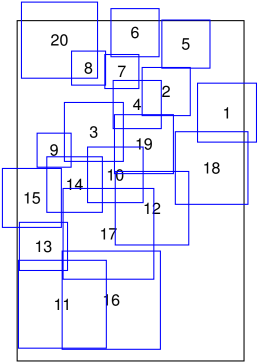
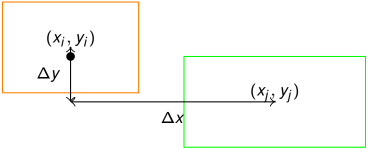


Figure: $\alpha = 0.2$

Recovering the No-Overlap Decisions



If $\Delta x \geq \Delta y$, then horizontal separation is enforced:

- If $x_i < x_j$, then department i is placed to the left of department j, i.e., the constraint

$$|x_i - x_j| \geq \frac{1}{2}(w_i + w_j) \text{ or } |y_i - y_j| \geq \frac{1}{2}(h_i + h_j)$$

  is replaced by the constraint

$$x_j - x_i \geq \frac{1}{2}(w_i + w_j).$$

- If $x_i < x_j$, then department i is placed to the right of department j.

If $\Delta y > \Delta x$, then vertical separation is enforced.

Second Stage Model

$$
\begin{aligned}
\min_{x_i, y_i, h_i, w_i} \quad & \sum_{1 \leq i < j \leq N} c_{ij}(u_{ij} + v_{ij}), \\
\text{s.t.} \quad & u_{ij} \geq x_j - x_i, \quad u_{ij} \geq x_i - x_j, \quad 1 \leq i < j \leq N, \\
& v_{ij} \geq y_j - y_i, \quad v_{ij} \geq y_i - y_j, \quad 1 \leq i < j \leq N, \\
& x_i + \frac{1}{2} w_i \leq \frac{1}{2} w_F, \quad \text{and} \quad x_i - \frac{1}{2} w_i \geq -\frac{1}{2} w_F, \; i = 1, \ldots, N, \\
& y_i + \frac{1}{2} h_i \leq \frac{1}{2} h_F, \quad \text{and} \quad y_i - \frac{1}{2} h_i \geq -\frac{1}{2} h_F, \; i = 1 \ldots N, \\
& w_i h_i \geq a_i, \quad i = 1, \ldots, N, \\
& \beta w_i - h_i \geq 0, \quad i = 1, \ldots, N, \\
& \beta h_i - w_i \geq 0, \quad i = 1, \ldots, N. \\
& w_i^{\min} \leq w_i \leq w_i^{\max}, \quad i = 1, \ldots, N, \\
& h_i^{\min} \leq h_i \leq h_i^{\max}, \quad i = 1, \ldots, N, \\
& \text{No-overlap constraints for all } 1 \leq i < j \leq N.
\end{aligned}
$$

Example of Second Stage Solution

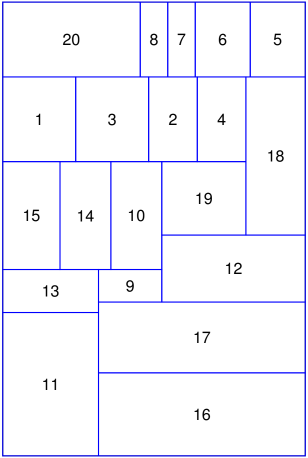


Figure: AB20 best solution with aspect ratio 3

Cost Improvement vs Number of α Values Used

Results for AB20 with varying number of α values used:

Number of α-values	Best Cost	Time	Line-by-line Improvement	Cumulative Improvement
20	3016.3	16.4 s	—	—
100	2939.0	73.4 s	2.6%	2.6%
500	2858.5	370.2 s	2.7%	5.2%
1000	2858.5	759.0 s	0.0%	5.2%
1500	2829.2	1074.5 s	1.0%	6.2%
2000	2806.4	1411.3 s	0.8%	7.0%

Test Instances

Instance	# of depts	Height	Width	Empty space	Flow density	Source
AB20	20	30	20	0.0%	64.7%	Armour & Buffa (1963)
SC30	30	15	12	9.4%	11.5%	Liu & Meller (2007)
Tam30	30	45	40	11.1%	67.4%	Tam (1992)
JLAV30-A	30	14	13	0.0%	75.9%	Jankovits et al. (2011)
JLAV30-B	30	20	10	0.0%	72.4%	Jankovits et al. (2011)
SC35	35	16	15	20.0%	9.1%	Liu and Meller (2007)
AnVi-50	50	21	18	1.6%	29.6%	New
AnVi-70	70	27	20	1.9%	29.1%	New
AnVi-80	80	26	22	0.3%	30.3%	New
AnVi-100	100	31	25	4.3%	31.3%	New

Implementation

- The computational tests were performed on a dual core Intel(R) Xeon(R) X5675 @ 3.07 GHz with 8 Gb of memory.
- The first stage model was solved using the nonlinear optimization solver SNOPT 7.2-8.
- The second stage model was solved with CPLEX 12.5.1.0.
- Both solvers were accessed using the modeling language AMPL.

Results for AB20

Aspect ratio	Jankovits et al.	K.-K. & Konak	Our approach	Cost reduction
10	——	3 758.7	2 793.7	25.7%
9	——	——	2 869.1	——
8	3 014.2	——	2 842.6	5.7%
7	2 979.3	4 718.8	2 829.3	5.0%
6	2 708.0	——	2 781.3	-2.7 %
5	3 009	5 023.7	2 858.5	5.0 %
4	2 960.5	5 196.3	2 919.1	1.4%
3	——	5 400.0	2 899.8	46.3%
2		—		

Results for Tam30

Aspect ratio	Kim et al. (1998)	Jankovits et al.	K.-K. & Konak	Our approach	Cost reduction
10	—	24 098	—	20 489.8	15.0%
9	—	23 924	—	20 391.8	14.8 %
8	—	23 420	—	20 514.1	12.4 %
7	—	23 974	—	20 505.0	14.5 %
6	—	23 770	—	20 528.6	13.6 %
5	—	24 916	19 009.90	20 523.8	-8.0 %
4	—	25 000	—	20 658.9	17.4 %
3	—	—	—	20 751.6	—
2	21 560.6	—	—	20 745.2	3.8%

Results for SC30 and SC35

Instance	Aspect ratio	Liu & Meller	K.-K. & Konak	Our approach	Cost reduction
SC30	5	3 706.83	3 370.98	4 342.8	-28.8%
	4	4 165.83	——	4 363.2	-4.7%
	3	4 332.87	——	4 564.2	-5.3%
	2	4 790.43	——	5 413.7	-13.0 %
SC35	5	3 247.48	——	3 655.6	-12.6%
	4	3 604.12	3 385.48	3 770.6	-11.4%
	3	4 332.87	——	3 999.0	7.7%
	2	4 839.45	——	4 808.6	0.6%

Results for JLAV30-A and JLAV30-B

JLAV30-A

Aspect ratio	Jankovits et al.	Our approach	Cost reduction
10	9445	8699.8	7.9%
9	9591	8510.1	11.3%
8	9312	8401.4	9.8%
7	9320	8471.7	9.1%
6	9504	8733.6	8.1%
5	9544	8577.4	10.1%
4	9509	8770.2	7.8%

JLAV30-B

Aspect ratio	Jankovits et al.	Our approach	Cost reduction
10	10511	9539.6	9.2%
9	10532	9771.2	7.2%
8	10506	9762.6	7.1%
7	10414	9768.1	6.2%
6	10604	9671.0	8.8%
5	10424	9930.5	4.7%
4	10199	9843.9	3.5%

Results for New Very Large Instances

Costs of the best layouts computed by our approach:

Aspect ratio	AnVi-50	AnVi-70	AnVi-80	AnVi-100
6	17714.2	42902.4	63717.4	117493.5
5	17727.0	43432.1	63744.1	117791.9
4	17927.4	42927.5	64509.2	117253.6
Time for 10 αs	70 s	150 s	230 s	760 s

Conclusion

We presented an improved version of the framework of Jankovits et al.:

- Improved first stage:
  - The departments are handled as rectangles.
  - Aspect ratio constraints are enforced exactly in the first stage.
  - Simpler objective function $\rightarrow$ improved computational times.
- Same second stage (second-order conic optimization problem).
- Normally improves on the results of Jankovits et al.
- Can compute solutions for instances with up to 100 departments in minutes.

Conclusion

We presented an improved version of the framework of Jankovits et al.:

- Improved first stage:
  - The departments are handled as rectangles.
  - Aspect ratio constraints are enforced exactly in the first stage.
  - Simpler objective function $\rightarrow$ improved computational times.
- Same second stage (second-order conic optimization problem).
- Normally improves on the results of Jankovits et al.
- Can compute solutions for instances with up to 100 departments in minutes.

Thank you for your attention.